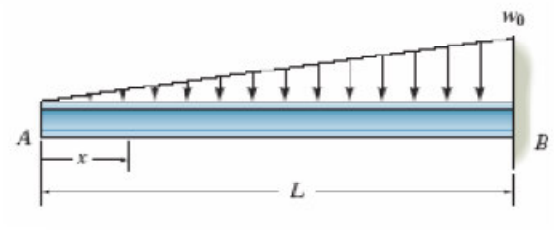
Problem 12-28

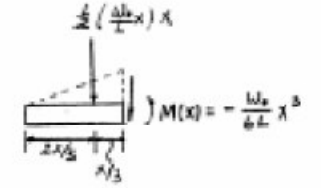
Determine the elastic curve for the cantilevered beam using the x coordinate. Also determine the maximum slope and maximum deflection. EI is constant.



$$EI\frac{d^2v}{dx^2} = M(x); \qquad EI\frac{d^2v}{dx^2} = -\frac{w_0x^3}{6L}$$

$$EI\frac{dv}{dx} = -\frac{w_0x^4}{24L} + C_1 \qquad (1)$$

$$EIv = -\frac{w_0x^5}{120L} + C_1x + C_2 \qquad (2)$$



Boundary conditions:

$$\frac{dv}{dx} = 0 \text{ at } x = L$$

From Eq. (1),

$$0 = -\frac{w_0}{24L}(L^4) + C_1; \qquad C_1 = \frac{w_0L^3}{24}$$

$$v = 0 \text{ at } x = L$$

From Eq. (2),

$$0 = -\frac{w_0}{120L}(L^5) + \frac{w_0L^3}{24}(L) + C_2; \qquad C_2 = -\frac{w_0L^4}{30}$$

The slope:

From Eq. (1),

$$\frac{dv}{dx} = \frac{w_0}{24EIL}(-x^4 + L^4)$$

$$\theta_{max} = \left.\frac{dv}{dx}\right|_{x=0} = \frac{w_0L^3}{24EI} \qquad \textbf{Ans}$$

The elastic curve:

From Eq. (2),

$$v = \frac{w_0}{120EIL}(-x^5 + 5L^4x - 4L^5) \qquad \textbf{Ans}$$

$$v_{max} = v\Big|_{x=0} = \frac{w_0L^4}{30EI} \qquad \textbf{Ans}$$