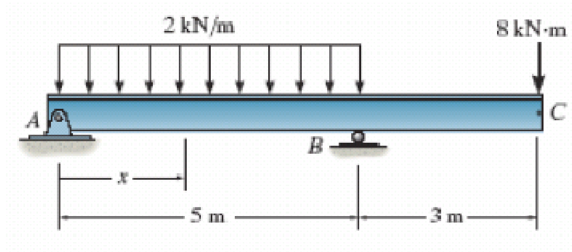
Problem 12-47

The beam is subjected to the load shown. Determine the slope at *A* and the displacement at *C*. *EI* is constant.



Use Macaulay Function: $\Psi(z) := \Phi(z)\cdot(z)$

Given: $a := 5m \qquad b := 3m$

$P := 8kN \qquad w := 2\dfrac{kN}{m}$

Solution: $L := a + b$

Support Reactions :

$+\uparrow \Sigma F_y = 0; \qquad A + B - w\cdot a - P = 0 \qquad$ **(1)**

$\circlearrowleft + \Sigma M_A = 0; \qquad (w\,a)\cdot(0.5a) - B\cdot a + P\cdot L = 0 \qquad$ **(2)**

Solving Eqs. (1) and (2): $B := \dfrac{1}{a}\cdot[(w\,a)\cdot(0.5a) + P\cdot L] \qquad A := w\cdot a + P - B$

$B = 17.8\,kN \qquad A = 0.2\,kN$

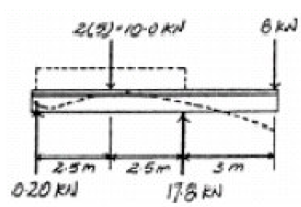
Moment Function :

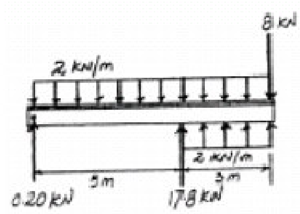
$$M(x) = -0.5w\cdot\Psi(x-0)^2 - (-0.5)w\cdot\Psi(x-a)^2 + A\cdot\Psi(x-0) + B\cdot\Psi(x-a)$$

$$M(x) = -0.5w\cdot x^2 + 0.5w\cdot\Psi(x-a)^2 + A\cdot x + B\cdot\Psi(x-a)$$

Slope and Elastic Curve : $EI := kN\cdot m^2$

$EI_o := 1$





$$EI\cdot\frac{d^2\cdot v}{dx^2} = M(x)$$

$$EI\cdot\frac{d^2v}{dx^2} = -0.5w\cdot x^2 + 0.5w\cdot\Psi(x-a)^2 + A\cdot x + B\cdot\Psi(x-a)$$

$$EI\cdot\frac{dv}{dx} = -\frac{1}{6}w\cdot x^3 + \frac{1}{6}w\cdot\Psi(x-a)^3 + \frac{A}{2}\cdot x^2 + \frac{B}{2}\cdot\Psi(x-a)^2 + C_1 \qquad (3)$$

$$EI\cdot v = -\frac{1}{24}w\cdot x^4 + \frac{1}{24}w\cdot\Psi(x-a)^4 + \frac{A}{6}\cdot x^3 + \frac{B}{6}\cdot\Psi(x-a)^3 + C_1\cdot x + C_2 \qquad (4)$$

Boundary Conditions : v=0 at x=0 and x=a. From Eq. (4):

$$0 = -0 + 0 + 0 + 0 + 0 + C_2 \qquad C_2 := 0$$

$$0 = -\frac{1}{24}w\cdot a^4 + 0 + \frac{A}{6}\cdot a^3 + 0 + C_1\cdot(a) \qquad C_1 := \frac{1}{24}w\cdot a^3 - \frac{A}{6}\cdot a^2 \qquad C_1 = 9.583\,kN\cdot m^2$$

Equation of Elastic Curve and slope :

$$v(x) := \frac{1}{EI}\left(-\frac{1}{24}w\cdot x^4 + \frac{1}{24}w\cdot\Psi(x-a)^4 + \frac{A}{6}\cdot x^3 + \frac{B}{6}\cdot\Psi(x-a)^3 + C_1\cdot x\right)$$

Displacement at *C*: Substitute x=L into Eq.(4). $v(L) = -160.75\dfrac{m}{EI_o}$ **Ans**

$$\theta(x) := \frac{1}{EI}\left(-\frac{1}{6}w\cdot x^3 + \frac{1}{6}w\cdot\Psi(x-a)^3 + \frac{A}{2}\cdot x^2 + \frac{B}{2}\cdot\Psi(x-a)^2 + C_1\right)$$

Slope at *A*: Substitute x=0 into Eq.(3). $\theta(0) = 9.58\dfrac{1}{EI_o}$ **Ans**