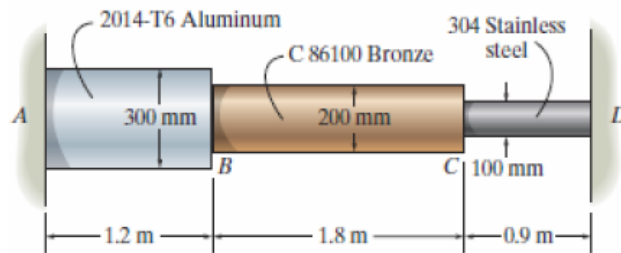


Problem 4-72

The assembly has the diameters and material make-up indicated. If it fits securely between its fixed supports when the temperature is $T_1 = 20^\circ\text{C}$, determine the average normal stress in each material when the temperature reaches $T_2 = 40^\circ\text{C}$.

Unit used: $^\circ\text{C} := \text{deg}$

Given: $T_1 := 20^\circ\text{C}$ $T_2 := 40^\circ\text{C}$
 $L_1 := 1.2\text{m}$ $d_1 := 300\text{mm}$
 $L_2 := 1.8\text{m}$ $d_2 := 200\text{mm}$
 $L_3 := 0.9\text{m}$ $d_3 := 100\text{mm}$



$\alpha_1 := 23 \cdot (10^{-6}) \frac{1}{^\circ\text{C}}$ $\alpha_2 := 17 \cdot (10^{-6}) \frac{1}{^\circ\text{C}}$ $\alpha_3 := 17 \cdot (10^{-6}) \frac{1}{^\circ\text{C}}$
 $E_1 := 73.1\text{GPa}$ $E_2 := 103\text{GPa}$ $E_3 := 193\text{GPa}$

Solution: $\Delta T := T_2 - T_1$ $\Delta T = 20^\circ\text{C}$

$$A_1 := \left(\frac{\pi}{4}\right) \cdot d_1^2 \quad A_2 := \left(\frac{\pi}{4}\right) \cdot d_2^2 \quad A_3 := \left(\frac{\pi}{4}\right) \cdot d_3^2$$

Equations of equilibrium:

$$\rightarrow \Sigma F_x = 0; \quad F_A - F_D = 0 \quad F_A = F_D$$

Let $F_A = F$. Then, $F_A = F_{AB} = F_{BC} = F_{CD} = F_D = F$

Thermal Expansion:

$$\delta_T := \alpha_1 \cdot (\Delta T) \cdot L_1 + \alpha_2 \cdot (\Delta T) \cdot L_2 + \alpha_3 \cdot (\Delta T) \cdot L_3 \quad \delta_T = 1.470000 \text{ mm}$$

Elongation: negative sign indicates shortening

$$\delta_F = -\frac{F \cdot L_1}{A_1 \cdot E_1} - \frac{F \cdot L_2}{A_2 \cdot E_2} - \frac{F \cdot L_3}{A_3 \cdot E_3}$$

Compatibility:

$$0 = \delta_T + \delta_F \quad \text{Given} \quad 0 = \delta_T - \frac{F \cdot L_1}{A_1 \cdot E_1} - \frac{F \cdot L_2}{A_2 \cdot E_2} - \frac{F \cdot L_3}{A_3 \cdot E_3} \quad [1]$$

Initial guess: $F := 1\text{kN}$ Solving [1]: $F := \text{Find}(F)$ $F = 1063.49 \text{ kN}$

Average Normal Stress: $\sigma_{al} := \frac{F}{A_1}$ $\sigma_{al} = 15.05 \text{ MPa}$ **Ans**

$\sigma_{br} := \frac{F}{A_2}$ $\sigma_{br} = 33.85 \text{ MPa}$ **Ans**

$\sigma_{st} := \frac{F}{A_3}$ $\sigma_{st} = 135.41 \text{ MPa}$ **Ans**